2701. Differentiate implicitly with respect to y, and set $\frac{dx}{dy} = 0$.

2702. Consider the possible directions of the alternative hypothesis, i.e. the possible directions of the tail.

2703. Assume, for a contradiction, that $y = \mathrm{f}(x)$ is cubic and convex everywhere. Show that $\mathrm{f}''(x)$ must be

① positive everywhere, and

② linear.

Find a contradiction from this.

2704. Consider the number of different ways of shading any two squares, and then the number of way in which they do share a border.

2705. Consider multiplicity for the roots of the equation for intersections $\mathrm{f}(x) - \mathrm{g}(x) = 0$.

2706. Set up an equation, using n and $n + 2$ as the odd numbers.

2707. Calculate the approximation using the quartic. Then the percentage error in the value V is given by $(V_{\text{approx}} - V_{\text{actual}})/V_{\text{actual}}$.

2708. Rewrite the logs on the RHS over base 8, and then combine them. Then exponentiate both sides over base 8.

2709. In both (a) and (b), just substitute values.

2710. This is possible if the individual sets form distinct subpopulations.

2711. Square both sides and sketch the resulting quartic, noting that the resulting curve will include new points not on the original curve.

2712. Take the usual area formula for a sector, and then differentiate it implicitly with respect to time. Substitute the arc length formula.

2713. (a) Consider the double root at $y = 12$.

(b) Find the intersection of the curves, and set up a single definite integral with respect to y to calculate $\frac{1}{2}A$.

2714. Differentiate implicitly with respect to x. Then set $\frac{dy}{dx} = 0$. Rearrange this, then substitute it back into the equation of the curve to show that $x \ln x + x - 1 = 0$. This is not analytically solvable. So, use the Newton-Raphson method.

2715. Use the binomial expansion to simplify the LHS.

2716. Find the mean value of the AP, which must be the third angle. Then work out the possible values for the smallest and largest angles and go from there.

2717. (a) There should be three forces on the load and four forces on the combined sledge and load.

(b) Resolve horizontally for the combined system.

(c) For total contact force, find the Pythagorean sum of the reaction and the friction.

2718. Solve for intersections, and look for double roots.

2719. Set aside the +1, which will simply add 4 to the eventual result. So, consider

$$\int_0^4 \frac{1}{8 - \sqrt{x}}\, dx.$$

Use the substitution $u = 8 - \sqrt{x}$. Find dx in terms of u and du. Remember to change the limits. That way, you don't need to convert back to x.

2720. Calculate the probability of seeing one of each colour, in terms of n. Set this equal to $\frac{16}{33}$, and solve for n.

2721. (a) Differentiate implicitly and rearrange.

(b) Find the gradient of the proposed normal, and take its negative reciprocal. Equate $\frac{dy}{dx}$ to this, and simplify. Then solve simultaneously with the hyperbola.

2722. If you can't visualise this in 3D, draw a 2D triangle. Find all three lengths using 3D Pythagoras on the points $(0, 0, 0)$, $(1, 1, 0)$, $(0, 1, 1)$.

2723. (a) Compare central tendency and spread.

(b) Give the sign and strength of any correlation.

(c) Locate these ten in the graph and consider them as a subpopulation.

2724. Differentiate by the quotient rule, and set the top to zero to find SPs. Then differentiate again by the quotient rule and evaluate the second derivative.

2725. In each case, make y the subject. Then consider the number of x values for which the numerator and denominator of the resulting fraction are equal to zero. The number of roots in the denominator corresponds to the number of vertical asymptotes.

2726. Use a double-angle formula. You can transform this into a standard integral and then write down the result.

2727. Multiply up by the denominator of the LHS. If you pick the correct coefficients to equate, then you'll get a contradiction immediately.

2728. (a) You should get a step!
   (b) Sketch $y = x^2$ and then multiply it by $\mathrm{S}(x)$.
   (c) Apply an input transformation to $y = \mathrm{S}(x)$.
   (d) Take the outputs of the step function, and feed them back in as inputs.

2729. Enact the differential operator, using the chain rule (differentiating implicitly).

2730. (a) Turn the information given into an equation. Equate to zero and complete the square for x.
   (b) Compare this to $x^2 + y^2 = 1$.

2731. Use the formula for the sum of an AP.

2732. At points of inflection, the second derivative is zero (and changes sign). Set up an equation and solve for p.

2733. (a) Consider moments around the centre (axis of symmetry) of the bale.
   (b) Draw a force diagram.

2734. Find $\frac{du}{dx}$ by the chain rule, then reciprocate and simplify using a double-angle formula.

2735. The acceptance region is the complement of the critical region.

2736. Look for common factors on the top and bottom before taking the limit.

2737. One is false.

2738. Add $2y$ to both sides first, then divide to separate the variables.

2739. Find the range of F, by completing the square or differentiating. Then consider G as as quadratic in x^2, and do likewise.

2740. Assume that $p > r$, and take p, q, r to be lengths in metres. Then set up two equations, one for vertical equilibrium and one for moments.

2741. This isn't true. Consider a counterexample when the individual sample means $(\bar{x}, \bar{y})$ are different.

2742. The graph is a part of a circle. Let $\arcsin x = \theta$.

2743. Call the numbers a and ϕa, with $a \neq 0$ and $\phi > 1$. Translate into algebra and solve a quadratic.

2744. For points of inflection, the second derivative must be zero, and must change sign. Use the chain rule to differentiate.

2745. (a) Consider the number of branches, and the probability of each individual branch.
   (b) There is no algebra needed.

2746. Integrate by parts: let $u = 2\ln t + 1$ and $v' = t$.

2747. (a) Consider a small positive value of t.
   (b) Solve $\frac{dV}{dt} = 0$.
   (c) Differentiate to find the maximum value of $\frac{dV}{dt}$.
   (d) Integrate $\frac{dV}{dt}$ between zero and infinity.

2748. Rewrite the numerator in polynomial terms of the denominator, and split the fraction up.

2749. Sketch the circles, looking for any intersections.

2750. For a conditioning method, consider the base first, in the case in which it is coloured red, and in the case in which it is not.

— Alternative Method —

For a combinatorial approach, there are 3^5 equally likely outcomes. Classify the successful outcomes by the number of red faces.

2751. (a) Use the fact that a cubic graph has rotational symmetry around its point of inflection.
   (b) Set the second derivative to zero.
   (c) Use the factor theorem.

2752. Consider the resultant force on the object. Also, consider the term "rigid".

2753. Find the range of $\cos^2 x$ first. Then decide if and how the input/output transformations given affect this range.

2754. Consider a^{LHS} and a^{RHS}.

2755. (a) Differentiate by the quotient rule.
   (b) Differentiate again, and evaluate the second derivative at the SPs.

2756. (a) Draw a force diagram for the pallet. Include a reaction force perpendicular to the slope and a frictional force parallel to it. Combine the two forces by Pythagoras to give the overall contact force.
   (b) Use trigonometry in the right-angled triangle from part (a).

2757. Note that, since h is a polynomial function, it can't have any asymptotes or discontinuities.

2758. To be well defined, each element of the domain must produce exactly one element of the codomain as its image. To be invertible, the mapping must be one-to-one.

Four are well defined, two are invertible.

2759. You know the first derivative of f. So, differentiate again by the product rule, using the result given in the first derivative.

2760. Assume, for a contradiction, that there exists a non-constant polynomial function f for which $\mathrm{f}(x) = \mathrm{f}(x+1)$ for all $x \in \mathbb{R}$. Consider the function g defined over $\mathbb{R}$ by

$$\mathrm{g}(x) = \mathrm{f}(x) - \mathrm{f}(0).$$

Show that this has infinitely many roots.

2761. (a) Find the coordinates of the endpoints.
(b) t_1 and t_2 are the t values at the end points. Multiply the integrand out before integrating.
(c) The parallel lengths a and b are y heights.

2762. (a) Consider the horizontal positions.
(b) Find the time to collision, in terms of d and θ, using a vertical *suvat*, then put this into the horizontal.

2763. (a) Rearrange. For ii, use the quadratic formula.
(b) Use the respective results from part (a).
(c) Differentiate the answer from (a) i. wrt y.
(d) Put all of the above together.

2764. Draw the possibility space as a 6×6 grid. Restrict it with the condition given, and use $p = \frac{\text{successful}}{\text{total}}$.

2765. Differentiate twice by the chain and product rules, and show that the second derivative is positive for all x.

2766. Draw a sketch. Assume, for a contradiction, that the object is in equilibrium. This means the three lines of action must be concurrent.

2767. Consider the graph in the form

$$y = \frac{|x|}{x} \times (4 + x).$$

The factor $\frac{|x|}{x}$ is a step function: $+1$ for positive x and -1 for negative x.

2768. Use log laws.

2769. Set the first derivative to zero. For the quartic to have no real roots, the stationary points must all have positive y coordinates.

2770. Consider the link between the negations of these statements.

2771. Integrate this by inspection: the integrand is the result of differentiation by the chain rule.

— Alternative Method —

You could use the substitution $u = x^2 + 4$, but it is significantly quicker to inspect.

2772. (a) One assumption concerns the strings, the other the pulleys and the strings.
(b) Consider the equation of motion along the strings.

2773. Rearrange to $y = \frac{1}{x^3}$, and compare the graph to that of $y = \frac{1}{x}$.

2774. (a) Exponentiate both sides over base k.
(b) Differentiate wrt y, and then reciprocate.

2775. The fact that the sum is equal to 0.5 gives you no information about the order of the results.

2776. Translate the entire problem by vector $-k\mathbf{j}$, then solve it, then translate it by vector $k\mathbf{j}$.

2777. The normal to $\triangle ABC$ passes through two of the vertices of the cube. In other words, it is a *space diagonal*. The normal to the base is vertical. Set up a 2D triangle involving these two lengths and use right-angled trig.

2778. Think coefficient of friction...

2779. Consider the boundary equations.

2780. "The quotient of two monic quadratic functions f and g" means $\mathrm{f}(x)/\mathrm{g}(x)$, where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are quadratics with leading coefficient 1.

Express the numerator $\mathrm{f}(x)$ as $\mathrm{g}(x) + ax + b$, then split the fraction up.

2781. Write the information above as a single equation, by equating values of the square of the common ratio.

2782. (a) Differentiate twice.
(b) Consider the velocity vector.

2783. Simplify $n^3 + (n+1)^3 + (n+2)^3$.

2784. (a) Use the binomial distribution.
(b) List the successful outcomes.

2785. Set the output to y, and rearrange to make x the subject. Then rewrite the instruction using x as the input value.

2786. If $y = \mathrm{f}(x)$ has rotational symmetry around the origin, then $\mathrm{f}(-x) = -\mathrm{f}(x)$ for all x. Differentiate this statement by the chain rule.

2787. The boundary equation is $X^2 + X = 10$.

2788. (a) Differentiate $A = xy$ using the product rule and substitute values.

(b) Consider the usage of the term "pixel" when referring to lengths and areas.

2789. Rearrange to polynomial form. Find a rational root using a polynomial solver or N-R. Take out the relevant factor, and show that the discriminant of the remaining quadratic is negative.

2790. Find a counterexample: four non-coplanar vectors which sum to zero. Be careful here, you need to be able to *show* that the four do not lie in any single plane.

2791. (a) Factorise.

(b) Use the factorisation in (a).

(c) Use a polynomial solver.

(d) Use the information of the previous parts.

2792. Sketch the curves.

2793. (a) Consider the parity (odd/evenness) of the number after each iteration.

(b) Work out how many $+1$'s and -1's are needed, then consider the number of different orders in which they could appear.

2794. (a) In each iteration, the number of line segments is quadrupled.

(b) K_n has $3 \times 4^{n-1}$ line segments. So, this is the number of triangles added to form K_{n+1}. Each of these has area $\left(\frac{1}{9}\right)^n$.

(c) Consider the total area added as the infinite sum of a GP.

2795. Reframe the problem with $t = 0$ at the moment the first cone is passed. Then set up two *suvat*s, one for the first 5 metres, the other for the first 10 metres, each in terms of u and a, the velocity at the first cone and the constant acceleration. Solve simultaneously.

2796. Newton-Raphson tends to be easiest.

2797. Use log rules to simplify top/bottom. Once you've got rid of a, split the fraction up and integrate.

2798. These two are not the same statement: (a) is a weaker statement than (b). However, that doesn't stop them both being true.

2799. Find x and y in terms of t. Then rearrange the horizontal equation to make t the subject, and sub into the vertical equation.

2800. The boundary equation is $\sqrt{6 - x^2} = x^2$. Solve this first, then sketch $y = \sqrt{6 - x^2}$ and $y = x^2$.

——— End of 28th Hundred ———